Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

What does it mean for an ordered pair to be a solution to a system of equations, and how do substitution, elimination, and graphing each reveal the solution in a different way?

Lesson Overview

A system of two linear equations consists of two equations sharing the same two variables. A solution is an ordered pair (x, y) that satisfies both equations simultaneously.

Substitution · Elimination · Graphing

One Sol.	Consistent & independent: lines intersect at exactly one point.
Infinite	Consistent & dependent: equations describe the same line.
No Sol.	Inconsistent: lines are parallel and never meet.
Method 1	Substitution — solve one equation for one variable, then substitute into the other.
Method 2	Elimination — multiply equations to make coefficients opposite, then add to eliminate one variable.
Method 3	Graphing — graph both lines and find the intersection (best for estimating, not exact answers).

Worked Examples

EXAMPLE 1

Solve by substitution: 2x + y = 7 and x − y = 2

- Solve the second equation for y: y = x − 2
- Substitute into the first equation: 2x + (x−2) = 7 → 3x−2=7 → 3x=9 → x=3
- Back-substitute: y = 3−2 = 1. Check: 2(3)+1=7 ✓ and 3−1=2 ✓

Answer: (3, 1)

EXAMPLE 2

Solve by elimination: 3x + 2y = 12 and 5x − 2y = 4

- The y-coefficients are already opposites (+2 and −2)
- Add the equations: (3x+5x)+(2y−2y) = 12+4 → 8x=16 → x=2
- Substitute: 3(2)+2y=12 → 2y=6 → y=3. Check: 5(2)−2(3)=4 ✓

Answer: (2, 3)

EXAMPLE 3

Solve by elimination (multiply first): 3x + 2y = 7 and x + 4y = 9

- Multiply the first equation by 2: 6x + 4y = 14
- Subtract the second equation: (6x−x)+(4y−4y) = 14−9 → 5x=5 → x=1
- Substitute into x+4y=9: 1+4y=9 → y=2. Check: 3(1)+2(2)=7 ✓

Answer: (1, 2)

EXAMPLE 4

Identify the system type: 2x − 4y = 6 and x − 2y = 3

- Divide the first equation by 2: x − 2y = 3
- This is identical to the second equation
- Both equations represent the same line → dependent system

Answer: Dependent — infinitely many solutions. Solution set: {(x,y) | x−2y=3}

EXAMPLE 5

Break-even: fixed costs $500, variable costs $8/unit, revenue $13/unit. Find break-even quantity

- Cost equation: C = 500 + 8x. Revenue equation: R = 13x
- Set equal for break-even: 500 + 8x = 13x
- Solve: 500 = 5x → x = 100. Verify: C=500+800=$1,300; R=13(100)=$1,300 ✓

Answer: Break-even at 100 units

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Solve by substitution: y = 3x − 1 and 2x + y = 9.

Hint: y is already isolated — substitute directly into the second equation.

2 Solve by elimination: 4x + 3y = 10 and 2x − 3y = 8.

Hint: The y-coefficients are already opposites — add the equations directly.

3 Solve: 5x + 2y = 16 and 3x − 4y = 6.

Hint: Multiply the first equation by 2 to make the y-coefficients opposites.

4 Determine the system type: $6x - 9y = 12$ and $2x - 3y = 5$.

Hint: Try to make the coefficients match — what happens?

5 A store sells adult ($12) and child ($8) tickets. 200 tickets were sold for $2,000. How many of each?

Hint: Set up two equations — one for total tickets, one for total revenue.

Key Vocabulary

System of Equations	Two or more equations with the same variables; the solution satisfies all equations simultaneously.
Consistent System	A system with at least one solution.
Inconsistent System	A system with no solution; the lines are parallel.
Dependent System	A system with infinitely many solutions; the equations represent the same line.
Substitution Method	Solving one equation for a variable and substituting into the other equation.
Elimination Method	Adding multiples of equations together to cancel (eliminate) one variable.
Break-Even Point	The quantity at which total cost equals total revenue.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 **Which ordered pair is a solution to the system $x + y = 5$ and $2x - y = 1$?** Multiple Choice

A (1, 4)
B (2, 3)
C (3, 2)
D (4, 1)

2 **Two lines are parallel. The system is:** Multiple Choice

A Consistent and independent
B Consistent and dependent
C Inconsistent
D Underdetermined

3 **Solve by elimination: 3x + y = 7 and x − y = 1.** Multiple Choice

A (1, 4)
B (2, 1)
C (3, −2)
D (4, −5)

4 **The system 2x − 4y = 8 and x − 2y = 4 is:** Multiple Choice

A Inconsistent
B Consistent and independent
C Consistent and dependent
D None of the above

5 **A system has exactly one solution. The lines:** Multiple Choice

A Are parallel
B Are the same line
C Intersect at one point
D Have no relationship

Common Mistakes to Avoid

✗ Substituting back into the same equation you solved for the variable.

✓ After finding x, substitute into the OTHER original equation to find y. Using the same equation gives a tautology.

✗ Forgetting to multiply BOTH sides when scaling an equation for elimination.

✓ If you multiply an equation by a constant, every term on both sides must be multiplied.

✗ Declaring a system inconsistent when it is actually dependent.

✓ If you get 0=0, the system is dependent (infinitely many solutions). If you get 0=nonzero, it is inconsistent (no solution).

✗ Stopping after finding x without solving for y.

✓ A solution to a 2-variable system is an ordered pair (x,y). Always find both values.

Math Tips

★ Choose substitution when one variable is already isolated (e.g., y=3x+1). Choose elimination when coefficients are easy to match.

★ Always check your solution by substituting back into BOTH original equations.

★ For word problems: define variables clearly, write two equations, then solve the system.

★ If elimination gives 0=0, the system is dependent. If it gives 0=k (k≠0), it is inconsistent.

★ Graphing is great for visualizing but gives only approximate answers. Use algebra for exact solutions.